

# Chiral Anomaly Effects and the BaBar Measurements of the $\gamma\gamma^* \rightarrow \pi^0$ Transition Form Factor

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The recent BaBar measurements of the  $\gamma\gamma^* \rightarrow \pi^0$  transition form factor show spectacular deviation from perturbative QCD prediction for large space-like  $Q^2$  up to  $34 \text{ GeV}^2$ . When plotted against  $Q^2$ ,  $Q^2 F(Q^2)$  shows steady increase with  $Q^2$  in contrast with the flat  $Q^2$  behavior predicted by perturbative QCD, and at  $34 \text{ GeV}^2$  is more than 50% larger than the QCD prediction. Stimulated by the BaBar measurements, we revisit our previous paper on the cancellation of anomaly effects in high energy processes  $Z^0 \rightarrow \pi^0\gamma$ ,  $e^+e^- \rightarrow \pi^0\gamma$  and apply our results to the  $\gamma^*\gamma \rightarrow \pi^0$  transition form factor measured in the  $e^+e^- \rightarrow e^+e^-\pi^0$  process with one highly virtual photon. We find that, the transition form factor  $F(Q^2)$  behaves as  $(\frac{m^2}{Q^2}) \times (\ln(Q^2/m^2))^2$  and produces a striking agreement with the BaBar data for  $Q^2 F(Q^2)$  with  $m = 132 \text{ MeV}$  which also reproduces very well the CLEO data at lower  $Q^2$ .

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The  $\gamma^*\gamma \rightarrow \pi^0$  transition form factor at large momentum transfer  $Q^2$  which could be measured in  $Z^0 \rightarrow \pi^0\gamma$  decay, in high energy  $e^+e^- \rightarrow \pi^0\gamma$  or in  $e^+e^- \rightarrow e^+e^-\pi^0$  collisions [1] where one of the photon is highly virtual has been the subject of studies using the quark parton picture of hadrons for hard exclusive processes since the earlier days of perturbative QCD. The interest in this transition form factor lies in the fact that it is one of the simplest quantities to compute in QCD and relatively easy to measure. At  $Q^2 = 0$ , it is given by the two-photon  $\pi^0$  decay governed by the Adler-Bell-Jackiw triangle chiral anomaly [2–4] which gives correctly the decay rate. At large  $Q^2$ , short-distance operator expansion(OPE) [5] or perturbative QCD [6–8] predicts  $F(Q^2) \sim 2 f_\pi/Q^2$  (we use the convention  $f_\pi = 93 \text{ MeV}$  in this paper). The earlier CLEO data [9] give values for  $F(Q^2)$  up to  $Q^2 = 8 \text{ GeV}^2$  somewhat below the perturbative QCD(pQCD) prediction, though, with a possible rise for  $Q^2 F(Q^2)$  above  $2.5 \text{ GeV}^2$ . Recently, the BaBar Collaboration has produced measurements for the transition form factor from 4 to  $34 \text{ GeV}^2$  [10] which show spectacular deviation from the perturbative QCD prediction as seen from the data for  $Q^2 F(Q^2)$  which rise steadily with  $Q^2$  in contrast with the rather flat behavior predicted by pQCD and is more than 50% above the QCD

prediction at  $34 \text{ GeV}^2$ . There are also measurements by CELLO [11] up to  $2.5 \text{ GeV}^2$  which are shown in [9, 10]. As mentioned in [10, 12], recent calculations [13, 14] using the light-cone sum rules method at next-to-leading order with various forms for the pion distribution amplitude, seem to obtain values for the transition form factor higher than the asymptotic limit of [6, 7], but with very different  $Q^2$  behavior than the BaBar data for  $Q^2 < 15 \text{ GeV}^2$  and are below the BaBar data for  $Q^2$  in the range from 20 to  $40 \text{ GeV}^2$  [10, 12]. A more recent work [15] seems to obtain results consistent with the BaBar data for  $Q^2 > 15 \text{ GeV}^2$  with a very broad pion distribution amplitude. Most of these calculations use the short-distance OPE of Lepage-Brodsky [6, 7] to obtain the pion distribution amplitude, but, in the case of the  $\gamma^* \rightarrow \gamma \pi^0$  transition form factor with one virtual photon with large  $q^2$ , the short-distance expansion parameter  $\omega = -2p \cdot q/q^2 = 1$  ( $p, q$  being the pion and virtual photon momentum, respectively), which is large and the calculation of Lepage and Brodsky for fixed but large  $Q^2$  cannot be trusted because the corrections are important as mentioned in [16].

Without further questioning the validity of the perturbative QCD prediction [17–19] which is based on the quark parton picture of the pion, one could already consider the role of chiral anomaly for processes involving a pion and highly virtual photons and the radiative decays of gauge bosons like  $Z^0 \rightarrow \pi^0 \gamma$  or  $W^\pm \rightarrow \pi^\pm \gamma$  decays [20]. In a previous paper [21, 22], we showed that, for these processes, from the modified PCAC equation due to the Adler-Bell-Jackiw anomaly, the quark-parton contribution to the axial current divergence given by the triangle graph cancels the anomaly term resulting in the suppression of the  $Z^0 \rightarrow \pi^0 \gamma$  decay amplitude as well as the  $\gamma^* \rightarrow \gamma \pi^0$  transition form factor measured in  $e^+e^- \rightarrow \pi^0 \gamma$ . We also showed that for  $Q^2$  large as in  $Z^0 \rightarrow \pi^0 \gamma$  decay or in  $e^+e^- \rightarrow \pi^0 \gamma$  process with a highly virtual photon, the amplitude behaves like  $(\ln(Q^2/m^2))^2/Q^2$ , and a similar behavior for space-like  $Q^2$ . The quantity  $Q^2 F(Q^2)$  is found to rise with  $Q^2$  as  $(\ln(Q^2/m^2))^2$ . This can be seen from the  $\ln(Q^2/m^2)/Q^2$  behavior at large  $Q^2$  of the absorptive part of the triangle graph contribution to the divergence of the axial vector current matrix element  $\langle \pi^0 | \partial_\mu A_\mu | \gamma^* \gamma \rangle$ . Hence the  $(\ln(Q^2/m^2))^2/Q^2$  behavior for the  $\gamma^* \gamma \rightarrow \pi^0$  transition form factor. This is one of the main differences between the chiral anomaly approach and the perturbative QCD quark parton approach to the  $\gamma^* \gamma \rightarrow \pi^0$  transition form factor. In the chiral anomaly approach, the triangle graph gives us the absorptive and the dispersive part, while the quark parton approach based on short-distance operator expansion gives us only the real part as given by the tree graph at the lowest order in perturbative QCD.

Our calculation of the Adler-Bell-Jackiw triangle anomaly contribution to the  $\gamma^* \gamma \rightarrow \pi^0$  transition form factor and similar calculations later on [23, 24] are done at a time when few data on

the transition form factors at large  $Q^2$  are available [9, 11]. Now that the Babar data are available over a large range of large momentum transfer  $Q^2$ , it is relevant to compare data with the anomaly contribution, considering the fact that, in pQCD the  $\gamma^*\gamma \rightarrow \pi^0$  transition form factor depends on the pion distribution which is not known to a good accuracy at present. For this reason, in this paper, we apply our previous analysis of the anomaly effects in  $Z^0 \rightarrow \pi^0\gamma$  to the  $\gamma^*\gamma \rightarrow \pi^0$  transition form factor at space-like  $Q^2$ . Using the generalized Goldberger-Treiman relation to relate the quark mass in the triangle graph to the quark-pion Yukawa coupling  $g$ , with  $m = g f_\pi$ , as in the linear sigma-model [25, 26], we show that the BaBar data can be reproduced with  $m = 132 \text{ MeV}$  or  $g = \sqrt{2}$ , consistent with the  $(\ln(Q^2/m^2))^2/Q^2$  behavior.

We begin by first recalling, for convenience, our derivation of the anomaly contribution to  $Z^0 \rightarrow \pi^0\gamma$  in [21]. Similar to the  $Z^0 \rightarrow \pi^0\gamma$  decay, the  $\gamma^*\gamma \rightarrow \pi^0$  amplitude,  $\mathcal{M} = \langle \pi^0(p) | T | \gamma^*(q) \gamma(k) \rangle$  in the reaction  $e^+e^- \rightarrow e^+e^-\pi^0$  has the form  $\epsilon_\mu(q)\epsilon_\nu(k)N^{\mu\nu}(q,k)$  with:

$$N^{\mu\nu}(q,k) = e^2 F(q,k) Y^{\mu\nu}, \quad Y^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} q_\alpha k_\beta. \quad (1)$$

where  $p$  is the produced pion momentum in the final state ( $p = q + k$ ),  $F(q,k)$  in the following, will be written as  $F(Q^2)$ , the form factor in the kinematic region of the BaBar measurement, with the virtual photon with momentum  $q$  space-like ( $q^2 = -Q^2 < 0$ ) while the photon with momentum  $k$  is almost on the mass-shell ( $k^2 \simeq 0$ ). As with the derivation of the two-photon  $\pi^0$  decay amplitude, we start with the modified PCAC equation due to the Adler-Bell-Jackiw triangle anomaly in the presence of the electromagnetic interactions. The divergence of the axial vector current associated with  $\pi^0$  becomes:

$$\partial_\mu A^\mu = f_\pi m_\pi^2 \phi + S \frac{e^2}{16\pi^2} \epsilon_{\alpha\beta\gamma\delta} F^{\alpha\beta} F^{\gamma\delta} \quad (2)$$

with  $F_{\mu\nu}$  the usual electromagnetic field strength tensor and  $S$  are the sum of the squares of the charges and colors of quark contributing to the anomaly, which takes the value  $S = 1/2$  [2]. Taking the matrix element of the l.h.s of Eq. (2), and separating the  $\pi^0$  pole term from the continuum, as previously shown [21], we arrive at the expression for the  $\gamma^*\gamma \rightarrow \pi^0$  amplitude:

$$N^{\mu\nu} = \frac{1}{f_\pi} \left( p_\tau \tilde{R}^{\mu\nu\tau}(q,k) - S \frac{e^2}{2\pi^2} Y^{\mu\nu} \right) \quad (3)$$

where  $\tilde{R}^{\mu\nu\tau}(q,k)$  is the triangle graph (the direct coupling between the three currents) or the continuum contribution to the axial vector current matrix element  $\langle 0 | A_\mu | \gamma^*\gamma \rangle$  defined as  $R^{\mu\nu\tau}(q,k)$ :

$$R^{\mu\nu\tau}(q,k) = \tilde{R}^{\mu\nu\tau}(q,k) - f_\pi \frac{p^\tau N^{\mu\nu}(q,k)}{p^2 - m_\pi^2} \quad (4)$$

As shown in [2, 27], gauge invariance and Bose symmetry tell us that the divergence  $p_\tau R^{\mu\nu\tau}(q, k)$  is in general proportional to  $q^2$  and  $k^2$  and does not vanish when one or both photons are off mass-shell. Only when both photons are real ( $q^2 = 0, k^2 = 0$ ) that  $p_\tau R^{\mu\nu\tau}(q, k)$  is  $O(p^2)$  and becomes negligible. One can then apply Eq. (3) to  $\pi^0 \rightarrow \gamma\gamma$  and finds that it is given by the anomaly [2–4]. For highly virtual photon as in the present  $\gamma^*\gamma \rightarrow \pi^0$  transition form factor, we will assume, as in [20], that  $\tilde{R}^{\mu\nu\tau}(q, k)$  is given by the triangle graph. From the expression given in [27], we find, assuming equal mass for  $u, d$  quarks in the triangle graph:

$$p_\tau \tilde{R}^{\mu\nu\tau}(q, k) = e^2 S \left( 2mP(q, k) + \frac{1}{2\pi^2} \right) Y^{\mu\nu} \quad (5)$$

where

$$P(q, k) = \frac{m}{2\pi^2} \int_0^1 dx \int_0^{1-x} \frac{dy}{D} \quad (6)$$

and

$$D = k^2 y(1-y) + q^2 x(1-x) - 2q \cdot kxy - m^2 \quad (7)$$

The quark mass  $m$  in the triangle graph is taken as a parameter to set the scale for the high energy limit, similar to the quark mass parameter used in the calculation of  $e^+e^- \rightarrow q\bar{q}$  processes at high energy, where the asymptotic limit is reached when  $Q^2 \gg m^2$  and the quark-parton picture is valid and the  $m^2/Q^2$  term in the cross section  $\sigma(e^+e^- \rightarrow \text{hadrons})$  can be neglected.

When both photons are real ( $q^2 = 0, k^2 = 0$ ), from Eq. (6), we get:

$$2mP(q, k) = -\frac{1}{2\pi^2} + O(p^2) \quad (8)$$

which implies that the r.h.s of Eq. (5) is  $O(p^2)$  in agreement with our previous remark that  $p_\tau \tilde{R}^{\mu\nu\tau}(q, k) = O(p^2)$ . For our transition form factor with time-like virtual photon  $Q^2 = s$ , with  $s > 4m^2$ , we have, as given in [21]:

$$2mP(q, k) = \frac{1}{2\pi^2} \left( \frac{m^2}{s} \right) K(m^2, s) \quad (9)$$

where

$$K(m^2, s) = \left( \ln \frac{1+\rho}{1-\rho} - i\pi \right)^2, \quad \rho = \sqrt{1 - 4m^2/s}, \quad s > 4m^2 \quad (10)$$

For space-like  $q$ , with  $q^2 = -Q^2$  ( $s = -Q^2$ ), with  $s < 0$ , by analytic continuation, the function  $K(m^2, s)$  becomes real and is given by:

$$K(m^2, Q^2) = \left( \ln \frac{\rho+1}{\rho-1} \right)^2, \quad \rho = \sqrt{1 + 4m^2/Q^2} \quad (11)$$

Using Eq. (9) for  $2mP(q, k)$  and Eq. (5) for the divergence  $p_\tau \tilde{R}^{\mu\nu\tau}(q, k)$ , we arrive at the final expression for  $\gamma^*\gamma \rightarrow \pi^0$  amplitude:

$$N^{\mu\nu} = \frac{1}{f_\pi} \frac{e^2}{2\pi^2} S \left( \frac{m^2}{Q^2} K(m^2, Q^2) \right) Y^{\mu\nu} \quad (12)$$

We note also that the term  $2mP(q, k)$  in Eq. (5) can be obtained directly from the triangle graph with the axial vector current replaced by the direct pion-quark vertex with the  $\gamma_5$  Yukawa coupling  $g = m/f_\pi$  as in the linear sigma model [25, 26] and PCAC is imposed on the pion-quark vertex. This is an equivalent method to obtain the  $\gamma^*\gamma \rightarrow \pi^0$  amplitude for large  $Q^2$  without having to go through the proof of anomaly cancellation [28]. The triangle graph with pion-quark vertex also gives us the term  $2mP(q, k)$  for the  $\pi^0 \rightarrow \gamma\gamma$  decay amplitude, as shown in Eq. (8).

For  $S = 1/2$ , the scalar transition form factor  $F(q, k)$  is given by:

$$F(q, k) = \frac{1}{f_\pi} \frac{1}{4\pi^2} \frac{m^2}{s} K(m^2, s) \quad (13)$$

and for the space-like  $\gamma^*\gamma \rightarrow \pi^0$  transition form factor, at large  $Q^2 \gg m^2$ , the dominant term in Eq. (13) is

$$F(Q^2) = \frac{1}{f_\pi} \frac{1}{4\pi^2} \frac{m^2}{Q^2} \left( \ln \frac{Q^2}{m^2} \right)^2 \quad (14)$$

to be compared with the transition form factor for real photon (our normalization is the same as in [29])

$$F(q^2 = 0, k^2 = 0, p^2 = 0) = - \left( \frac{1}{4\pi^2 f_\pi} \right) \quad (15)$$

We emphasize that our results Eqs. (9–15) are exact calculations and the result for the transition form factor given in Eq. (14) is valid for large  $Q^2$  ( $Q^2 \gg m^2$ ), including its  $Q^2 \rightarrow \infty$  limit.

This result, the  $(\ln(Q^2/m^2))^2$  rise for  $Q^2 F(Q^2)$  which has been obtained in [21], are obtained later in [23, 24]. Ref. [23] obtains the anomaly contribution from the PCAC equation for the matrix element of the axial vector current between the photons, while Ref. [24] equates the divergence of the axial vector current two-photon matrix element with its pion pole terms. In our calculation, the quark mass parameter is taken as a dynamical (constituent) quark mass. The reason why we take constituent quark mass is also emphasized in [23, 24]. In fact as argued in Ref. [23], because of the presence of the Nambu-Goldstone pion pole term in the PCAC equation, the divergence of the axial vector current should also be non-vanishing in the limit of vanishing current quark mass, to cancel the longitudinal term generated by the pion pole term, as in the derivation of the Goldberger-Treiman relation for the pion-nucleon coupling constant

As shown in Fig. (1), our prediction for the quantity  $Q^2 F(Q^2)$  for  $m = 132$  MeV fits very well the CLEO and BaBar data. The agreement with the BaBar data is striking, as our predicted transition form factor depends on only one parameter, the effective mass for quark in the triangle graph. We note that recent works with various models for the pion distribution mentioned above [13, 15] seem unable to obtain the rise of  $Q^2 F(Q^2)$  for  $Q^2 > 20$  GeV<sup>2</sup> as the BaBar measurements. Our value for the effective quark mass is consistent with the high energy behavior of the  $e^+e^- \rightarrow \text{hadrons}$  cross section for which the ratio  $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$  is constant for  $s$  above a few GeV<sup>2</sup>.

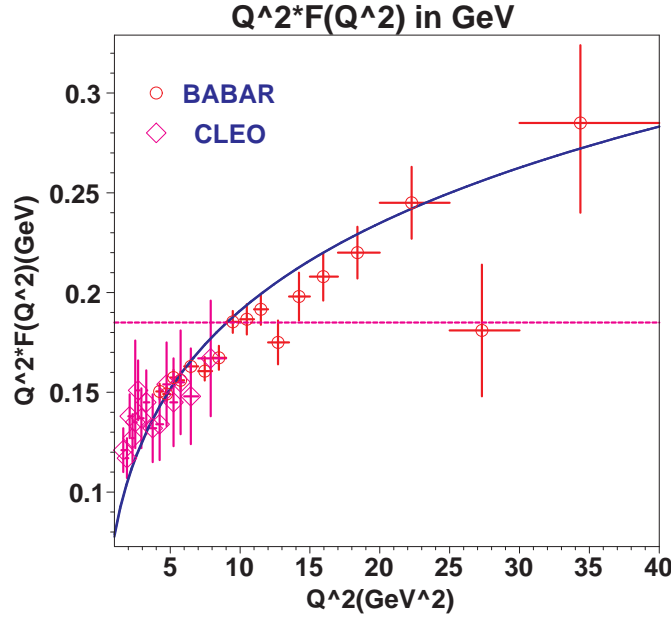


FIG. 1: Chiral anomaly prediction(solid line) for  $Q^2 F(Q^2)$  compared with the BaBar and CLEO measured values and the large  $Q^2$  pQCD prediction (horizontal dash line) of [6]

In conclusion, we have shown that chiral anomaly effects produce a  $(m^2/Q^2)(\ln(Q^2/m^2))^2$  behavior for the  $\gamma\gamma^* \rightarrow \pi^0$  transition form factor at  $Q^2 \gg m^2$  in contrast with the  $2f_\pi/Q^2$  behavior given by perturbative QCD. It is remarkable that our simple expression for the transition form factor is able to explain the CLEO and BaBar data. Our prediction for the rise of  $Q^2 F(Q^2)$  at higher space-like  $Q^2$  could be confirmed with further measurements. Similar behavior is expected for the time-like transition form factor and could be seen in  $e^+e^- \rightarrow \pi^0\gamma$ , as in  $e^+e^- \rightarrow \eta^{(\prime)}\gamma$  for which the time-like transition form factor has been measured by CLEO at  $Q^2 = 14.2$  GeV<sup>2</sup> [30] and is found to be close to the corresponding space-like values [12], and by BaBar at  $Q^2 = 112$  GeV<sup>2</sup> [31]. So, if chiral anomaly is indeed the cause of the rise like  $(\ln(Q^2/m^2))^2$  of  $Q^2 F(Q^2)$ , then, the pion might very well, for  $\gamma\gamma^* \rightarrow \pi^0$  process at high energies, behave like a Nambu-Goldstone boson,

like the Adler zero in low-energy  $\pi\pi$  scatterings and in  $\psi' \rightarrow J/\psi\pi\pi$  decay which are obtained from chiral symmetry constrains.

*Note added.* After the completion of this paper, we were informed of the papers by Dorokhov [32, 33] in which similar results are obtained using the triangle graph with pion-quark coupling given in [34].

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